Class X Session 2024-25
Subject - Mathematics (Basic)
Sample Question Paper - 8

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

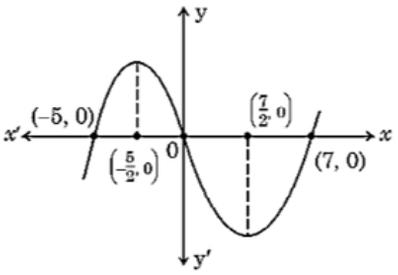
Section A

1. The ratio of HCF to LCM of the least composite number and the least prime number is: **[1]**

   a) 1 : 1 b) 2 : 1

   c) 1 : 2 d) 1 : 3

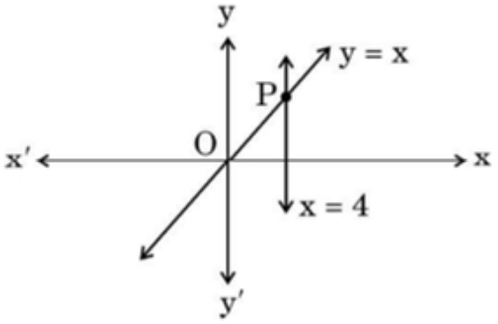
2. The graph of y = p(x) is given in the adjoining figure. Zeroes of the polynomial p(x) are **[1]**



   a) $-5, \frac{-5}{2}, \frac{7}{2}, 7$ b) -5, 7

   c) -5, 0, 7 d) $\frac{-5}{2}, \frac{-7}{2}$

3. The lines represented by the linear equations y = x and x = 4 intersect at P. The coordinates of the point P are: **[1]**



a) (4, 4)

b) (-4, 4)

c) (0, 4)

d) (4, 0)

4. The length of a rectangular field exceeds its breadth by 8 m and the area of the field is 240 m^2. The breadth of the field is **[1]**

a) 16 m

b) 30 m

c) 12 m

d) 20 m

5. The number of terms of an AP 5, 9, 13, …. 185 is: **[1]**

a) 41

b) 51

c) 31

d) 46

6. The distance between the points (3, -2) and (-3, 2) is: **[1]**

a) 40

b) $4\sqrt{10}$

c) $2\sqrt{10}$

d) $\sqrt{52}$

7. x-axis divides the line segment joining A(2, -3) and B(5, 6) in the ratio: **[1]**

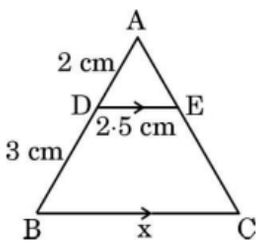
a) 2 : 1

b) 2 : 3

c) 3 : 5

d) 1 : 2

8. In the given figure, AD = 2 cm, DB = 3 cm, DE = 2.5 cm and DE || BC. The value of x is: **[1]**



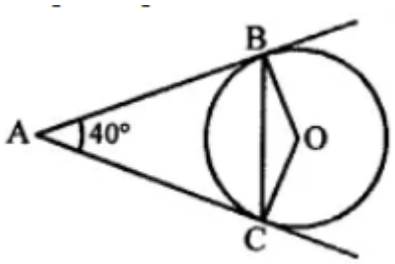
a) 7.5 cm

b) 3.75 cm

c) 6.25 cm

d) 6 cm

9. In the given figure, AB and AC are tangents to the circle with centre O such that ∠BAC = 40°. Then ∠BOC is equal to **[1]**



a) 140°

b) 120°

c) 80°

d) 100°

10. If angle between two radii of a circle is 130^o, the angle between tangents at ends of radii is : **[1]**

a) 70°

b) 90°

c) 60°

d) 50°

11. sec θ when expressed in terms of cot θ, is equal to: **[1]**

a) $\frac{\sqrt{1-\cot^2\theta}}{\cot\theta}$

b) $\frac{1+\cot^2\theta}{\cot\theta}$

c) $\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$

d) $\sqrt{1+\cot^2\theta}$

12. If cos A = $\frac{5}{8}$, then value of cot A·sin A is: **[1]**

a) $\frac{8}{5}$

b) $\frac{5}{8}$

c) $\frac{8}{\sqrt{39}}$

d) $\frac{5}{\sqrt{39}}$

13. The shadow of a 5 m long stick is 2 m long. At the same time, the length of the shadow of a 12.5 m high tree is **[1]**

a) 3 m

b) 4.5 m

c) 3.5 m

d) 5 m

14. The area of a sector of angle α (in degrees) of a circle with radius R is: **[1]**

a) $\frac{\alpha}{180} \times 2\pi R$

b) $\frac{\alpha}{180} \times \pi R^2$

c) $\frac{\alpha}{360} \times 2\pi R$

d) $\frac{\alpha}{360} \times \pi R^2$

15. The area of the sector of a circle of radius 10.5 cm is 69.3 cm^2. Find the central angle of the sector. **[1]**

a) 85^o

b) 72^o

c) 70^o

d) 26^o

16. A die is thrown once. Find the probability of getting a number less than 7. **[1]**

a) $\frac{1}{6}$

b) 0

c) $\frac{5}{6}$

d) 1

17. If three coins are tossed simultaneously, what is the probability of getting at most one tail? **[1]**

a) $\frac{3}{8}$

b) $\frac{5}{8}$

c) $\frac{7}{8}$

d) $\frac{4}{8}$

18. Mean and median of some data are 32 and 30 respectively. Using empirical relation, mode of the data is: **[1]**

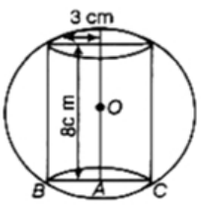
a) 36

b) 30

c) 20

d) 26

19. **Assertion (A):** In the given figure, a sphere circumscribes a right cylinder whose height is 8 cm and radius of the base is 3 cm. The ratio of the surface area of the sphere and the cylinder is 6 : 11 **[1]**



Reason (R): Ratio of their surface area = $\frac{Surface\ area\ of\ sphere}{Surface\ area\ of\ cylinder}$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The constant difference between any two terms of an AP is commonly known as common difference. **[1]**

Reason (R): The common difference of 2, 4, 6, 8 this A.P. is 2.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

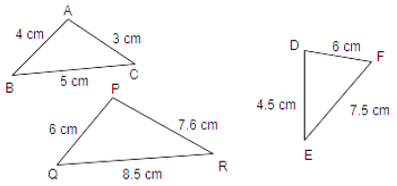
c) A is true but R is false.

d) A is false but R is true.

Section B

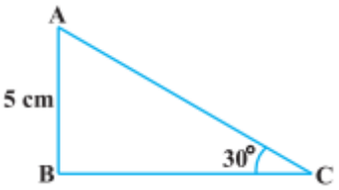
21. Can two numbers have 15 as their HCF and 175 as their LCM ? Give reasons. **[2]**

22. State which pairs of triangles in the given figure are similar? Also, state the similarity criterion used. **[2]**



23. From a point P, 10 cm away from the centre of a circle, a tangent PT of length 8 cm is drawn. Find the radius of the circle. **[2]**

24. In △ABC, right-angled at B, AB = 5 cm and ∠ACB = 30°. Determine the lengths of sides BC and AC. **[2]**



OR

Prove that (sin θ + cos θ) (tan θ + cot θ) = sec θ + cosec θ.

25. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the minor segment. [Use π = 3.14.] **[2]**

OR

Find the area of the minor and the major sectors of a circle with radius 6 cm, if the angle subtended by the minor arc at the centre is 60^o. (Use π = 3.14)

Section C

26. Prove that $(3+\sqrt{2})$ is irrational. **[3]**

27. One zero of the polynomial x^2- 2x - (7p + 3) is -1, find the value of p and the other zero. **[3]**

28. Find the value of a, b and c such that the numbers a, 7, b, 23 and c are in A.P. **[3]**

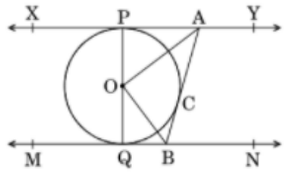
OR

If the m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} term be $\frac{1}{m}$, then show that its $(mn)^{th}$ term is 1.

29. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre. **[3]**

OR

In the figure, XY and MN are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and MN at B. Prove that ∠AOB = 90^o.



30. If cosec A + cot A = m. show that $\frac{m^2-1}{m^2+1}$ = cos A. **[3]**

31. Weekly income of 600 families is given below : **[3]**

Income (in Rs)	0 -1000	1000-2000	2000-3000	3000-4000	4000-5000	5000-6000
No. of Families	250	190	100	40	15	5

Find the median.

Section D

32. A piece of cloth costs 200 Rupees . If the piece was 5 m longer and each metre of cloth costs 2 Rupees less, the cost of the piece would have remain unchanged. How long is the piece and what is the original rate per metre? **[5]**

OR

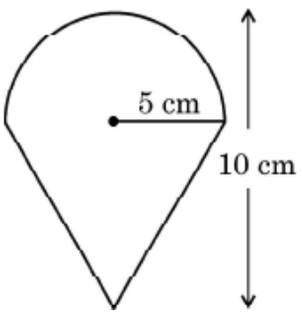
Two pipes together can fill a tank in $\frac{15}{8}$ hours. The pipe with larger diameter takes 2 hours less than the pipe with smaller diameter to fill the tank separately. Find the time in which each pipe can fill the tank separately.

33. From a point P on the ground, the angle of elevation of the top of a 10 m tall building is 30^o. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45^o. Find the length of the flagstaff and the distance of the building from the point P. (use $\sqrt{3}$ = 1.73) **[5]**

34. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder as shown in the figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article. **[5]**

OR

An ice-cream filled cone having radius 5 cm and height 10 cm is as shown in the figure. Find the volume of the ice-cream in 7 such cones.



35. The following table gives the marks obtained by 50 students in a class test: **[5]**

Marks	11 - 15	16 - 20	21 - 25	26 - 30	31 - 35	36 - 40	41 - 45	46 - 50
Number of students	2	3	6	7	14	12	4	2

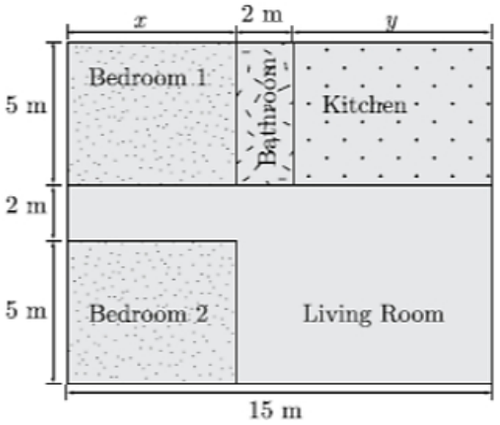
Calculate the mean and median for the above data.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Architect : An architect is a skilled professional who plans and designs buildings and generally plays a key role in their construction. Architects are highly trained in the art and science of building design. Since they bear responsibility for the safety of their buildings' occupants, architects must be professionally licensed.

Vishu is a licensed architect and design very innovative house. She has made a house layout for her client which is given below. In the layout, the design and measurements has been made such that area of two bedrooms and kitchen together is 95 sq. m.



i. Which pair of linear equations does describe this situation? (1)

ii. What is the length of the outer boundary of the layout? (1)

iii. What is the area of the bedroom 1? (2)

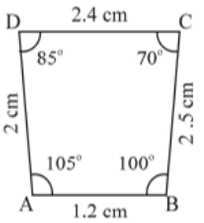
OR

What is the area of living room in the layout? (2)

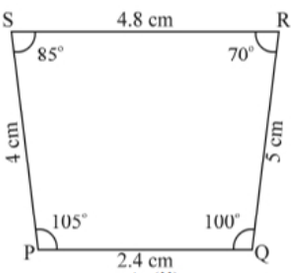
37. **Read the following text carefully and answer the questions that follow:** **[4]**

Observe the figures given below carefully and answer the questions:

Figure A

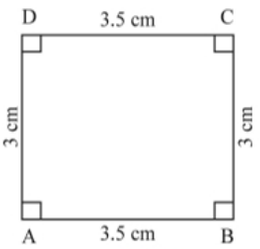


A (i)

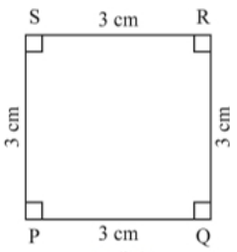


A (ii)

Figure B

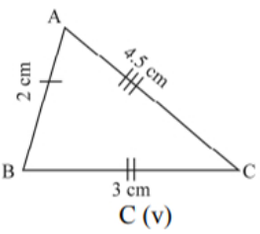


B (iii)

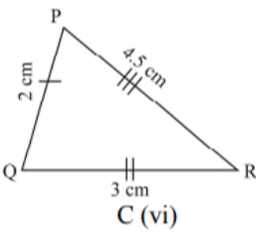


B (iv)

Figure C



C (v)



C (vi)

i. Name the figure(s) where in two figures are similar. (1)

ii. Name the figure(s) wherein the figures are congruent. (1)

iii. Prove that congruent triangles are also similar but not the converse. (2)

OR

What more is least needed for two similar triangles to be congruent? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

A satellite image of a colony is shown below. In this view, a particular house is pointed out by a flag, which is situated at the point of intersection of x and y-axes. If we go 2 cm east and 3 cm north from the house, then we reach to a Grocery store. If we go 4 cm west and 6 cm south from the house, then we reach to an Electricians's shop. If we go 6 cm east and 8 cm south from the house, then we reach to a food cart. If we go 6 cm west and 8 cm north from the house, then we reach a bus stand.

Scale:

x-axis : 1 cm = 1 unit

y-axis : 1 cm = 1 unit

i. What is the distance between the grocery store and food cart? (1)

ii. What is the distance of the bus stand from the house? (1)

iii. If the grocery store and electricians shop lie on a line, then what will be the ratio of distance of house from grocery store to that from electrician’s shop? (2)

OR

What are the ratio of distances of the house from bus stand to food cart? (2)

Solution

Section A

1.

(c) 1 : 2

Explanation: Least composite number is 4 and the least prime number is 2.

LCM (4, 2) = 4

HCF (4, 2) = 2

The ratio of HCF to LCM = 2 : 4 or 1 : 2.

2.

(c) -5, 0, 7

Explanation: The graph intersect the x-axis at three distinct Points -5, 0, 7. So, there are three zeroes of P(x) which are -5, 0, 7.

3. **(a)** (4, 4)

Explanation: (4, 4)

4.

(c) 12 m

Explanation: Let the breadth of the rectangular field be x m

Therefore Length of the rectangular field = (x + 8) m

Area of the rectangular field = 240 m^2 (Given)

$\therefore (x + 8) \times x = 240$ (Area = Length $\times$ Breadth)

$\Rightarrow x^2 + 8x - 240 = 0$

$\Rightarrow x(x + 20) - 12(x + 20) = 0$

$\Rightarrow (x + 20)(x - 12) = 0$

$\Rightarrow x + 20 = 0$ or x - 12 = 0

$\Rightarrow$ x = -20 or x = 12

$\therefore$ x = 12 (Breadth cannot be negative)

Thus the breadth of the field is 12 m.

5.

(d) 46

Explanation: n^{th} term of an AP ,a_n = a + (n − 1)d

5 + (n − 1)4 = 185

Hence, n = 46

Therefore number of terms are 46

6.

(d) $\sqrt{52}$

Explanation: Let us take (3, – 2) and (– 3, 2) as (x_1, y_1) and (x_2, y_2)

Using distance formjula, d = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

d = $\sqrt{(-3 - 3)^2 + (2 - (-2))^2}$

d = $\sqrt{(-6)^2 + (2 + 2)^2}$

d = $\sqrt{36 + (4)^2}$

d = $\sqrt{36 + 16}$

d = $\sqrt{52}$

7.

(d) 1 : 2

Explanation: Let the ratio is k : 1

So the coordinate of the point are

$\left[\left(\frac{5k+2}{k+1}\right), \left(\frac{6k-3}{k+1}\right)\right]$

Since the point lies on x-axis, it's y coordinate will be 0

Comparing the coordinates

$\frac{6k-3}{k+1} = 0$

6k - 3 = 0

6k = 3

k = $\frac{1}{2}$

Required ratio is

= $\frac{1}{2}$: 1

= 1 : 2

8.

(c) 6.25 cm

Explanation: In △ADE and △ABC

∠D = ∠B {Corresponding angle}

∠E = ∠C {Corresponding angle}

∴ △ADE and △ABC (by A A Similarity)

$\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{2}{5} = \frac{2.5}{X}$

X = $\frac{5\times2.5}{2} = \frac{12.5}{2}$

= 6.25 cm

9. **(a)** 140°

Explanation: In the given figure, AB and AC are tangents to the circle with centre O such that ∠BAC = 40°, ∠BOC = ?

AB and AC are tangents and OB and OC are radii.

OB ⊥ AB and OC ⊥ AC

⇒ ∠OBA = 90° and ∠OCA = 90°

In quadrilateral BOCA,Angle sum of all angles = 360°=∠OBA+∠OCA=180

Hence,

⇒ 40° + ∠BOC = 180°

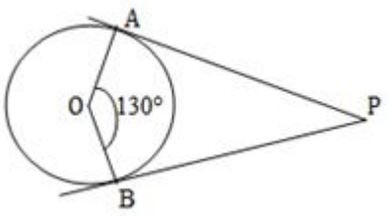
⇒ ∠BOC = 180° – 40° = 140°

10.

(d) 50°

Explanation: If the angle between two radii of a circle is 130^o, the angle between tangents at ends of radii is∠APB = 50°.

Because the angle between the two tangents drawn from an external point to a circle is supplementary of the angle between the radii of the circle through the point of contact.



11.

(c) $\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$

Explanation: As we know that,

$\sec^2\theta = 1 + \tan^2\theta$

and $\cot\theta = \frac{1}{\tan\theta}$

$\Rightarrow \tan\theta = \frac{1}{\cot\theta}$

$\therefore \sec^2\theta = 1 + \left(\frac{1}{\cot\theta}\right)^2$

$= 1 + \frac{1}{\cot^2\theta}$

$\Rightarrow \sec^2\theta = \frac{\cot^2\theta+1}{\cot^2\theta}$

$\Rightarrow \sec\theta = \frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$

12.

(b) $\frac{5}{8}$

Explanation: $\frac{5}{8}$

13.

(d) 5 m

Explanation: Ratio of lengths of objects = ratio of lengths of their shadows.

Let the length of shadow of the tree be x m. Then,

$\frac{5}{12.5} = \frac{2}{x} \Rightarrow$ 5x = 2 $\times$ 12.5 = 25

$\Rightarrow$ x = 5

14.

(d) $\frac{\alpha}{360} \times \pi R^2$

Explanation: $\frac{\alpha}{360} \times \pi R^2$

15.

(b) 72^o

Explanation: It is given that area of the sector = 69.3 cm^2

and Radius = 10.5 cm

Now, Area of the sector $= \frac{\pi r^2\theta}{360}$

$\Rightarrow \frac{\pi\times(10.5)^2\times\theta}{360}$ = 69.3

$\Rightarrow \theta = \frac{69.3\times360\times7}{10.5\times10.5\times22}$ = 72^o

Therefore, Central angle of the sector = 72^o

16.

(d) 1

Explanation: 1

17.

(d) $\frac{4}{8}$

Explanation: Three coins are tossed together

Hence,Total outcomes = {HHH, HHT, HTH, THH, HTT, TTH, TTT, THT} = 8

Favourable outcomes of getting atmost one tail = HHH, HHT, HTH, THH = 4

P(E) = $\frac{(Number\ of\ favourable\ outcomes)}{(Number\ of\ possible\ outcomes)}$ = $\frac{4}{8}$

18.

(d) 26

Explanation: mode = 3 median - 2 mean

= 3(30) - 2(32)

= 90 - 64

= 26

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21. $\frac{175}{15} = 11.667$

Hence 175 is not divisible by 15

But LCM of two numbers should be divisible by their HCF.

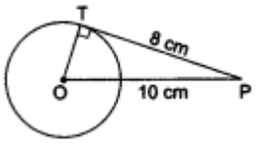
$\therefore$ Two numbers cannot have their HCF as 15 and LCM as 175.

22. Here, $\frac{AB}{DF} = \frac{4}{6} = \frac{2}{3}$, $\frac{BC}{EF} = \frac{5}{7.5} = \frac{2}{3}$, $\frac{AC}{DE} = \frac{3}{4.5} = \frac{2}{3}$

As, $\frac{AB}{DF} = \frac{BC}{EF} = \frac{AC}{DE}$

So, $\triangle ABC \sim \triangle DFE$ [by SSS similarity criterion]

Hence ABC and DFE are similar triangles, but no other pairs of triangles in the given figure are similar.

23. 

Let O be the centre of the given circle.

Then, OP = 10 cm. Also, PT = 8 cm.

Join OT.

Now, PT is a tangent at T and OT is the radius through the point of contact T.

$\therefore \quad OT \perp PT$

In the right ΔOTP

we have

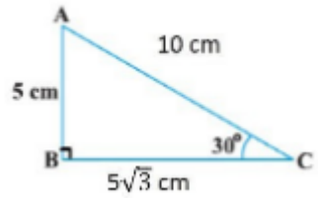
$OP^2 = OT^2 + PT^2$ [by Pythagoras' theorem]

$\Rightarrow$ OT = $\sqrt{OP^2 - PT^2} = \sqrt{(10)^2 - (8)^2}\text{cm} = \sqrt{36}\text{cm}$ = 6cm.

Hence, the radius of the circle is 6 cm.

24. Given AB = 5 cm

$\angle$ACB = 30^o



According to diagram,

tan C = $\frac{side\ opposite\ to\ angle\ C}{side\ adjacent\ to\ angle\ C}$

tan 30^o = $\frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{5}{BC}$

BC = $5\sqrt{3}$ cm

sin C = $\frac{side\ of\ angle\ C}{hypotenuse}$

sin 30^o = $\frac{AB}{AC}$

$\frac{1}{2} = \frac{5}{AC}$

AC = 10 cm.

OR

L.H.S. = (sin θ + cos θ)(tan θ + cot θ)

= (sin θ + cos θ) $\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

= (sin θ + cos θ) $\left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}\right)$

= (sin θ + cos θ) $\times \frac{1}{\sin\theta \cos\theta}$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$= \frac{\sin\theta + \cos\theta}{\cos\theta \sin\theta}$

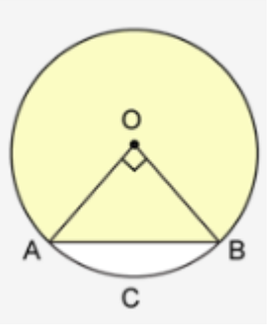
$= \frac{\sin\theta}{\cos\theta \sin\theta} + \frac{\cos\theta}{\cos\theta \sin\theta}$

$= \frac{1}{\cos\theta} + \frac{1}{\sin\theta}$

= sec θ + cosec θ

= R.H.S.

Hence proved.

25. 

Let O be the centre of the circle and AB be the chord.

Now, Area of the minor segment $= Area\ of\ the\ sector\ OACBO - Area\ of\ \triangle AOB$

$= 3.14 \times 10 \times 10 \times \frac{90}{360} - \frac{1}{2} \times 10 \times 10$

$= 78.5 - 50$

$= 28.5cm^2$

OR

Area of minor sector = $\frac{3.14 \times (6)^2 \times 60^\circ}{360^\circ}$

= 18.84

Hence, area of minor sector is 18.84 cm^2

Area of major sector = Area of circle - Area of minor sector

= 3.14 $\times$ $(6)^2$ - 18.84

= 94.2

Hence, area of major sector is 94.2 cm^2

Section C

26. If possible let $3 + \sqrt{2}$ is rational number, and we take another rational number 3 for our calculation.

$\Rightarrow (3 + \sqrt{2}) - 3 = \sqrt{2}$ (difference of two rational number is a rational number)

$\therefore \sqrt{2}$ is rational

This contradicts the fact that $\sqrt{2}$ is irrational

Since the contradiction arises by assuming that $3 + \sqrt{2}$ is rational.

Hence, $3 + \sqrt{2}$ is irrational.

27. Let p(x) = x^2 - 2x - (7p + 3)

Since -1 is a zero of p(x). Therefore,

p(-1) = 0

$(-1)^2$ - 2(-1) - (7p + 3)=0

1 + 2 - 7p - 3 = 0

3 - 7p - 3 = 0

7p = 0

p = 0

Thus, p(x) = x^2 - 2x - 3

For finding zeros of p(x), we put,

p(x) = 0

x^2 - 2x - 3 = 0

x^2 - 3x - x - 3 = 0

x(x-3) + 1(x-3) = 0

(x-3)(x+1) = 0

Put x - 3 = 0 and x + 1 = 0, we get,

Thus, x = 3, -1

Thus, the other zero is 3.

28. $a, 7, b, 23$ and c are in A.P.

Let the common difference be d.

$a + d = 7$ ....(i)

$a + 3d = 23$ ....(ii)

From (i) and (ii), we get

$2d = 16$

$d = 8$

Put d = 8 in (1) we get

$a + 8 = 7$

$a = -1$

$b = a + 2d$

$b = -1 + 2 \times 8$

or,$b = -1 + 16$

or, $b = 15$

$c = a + 4d$

= - 1 + 4 × 8

= - 1 + 32

$c = 31$

$\therefore a = -1, b = 15, c = 31$

OR

Let a and d be the first term and common difference respectively of the given A.P. Then

a_n = a + (n - 1)d

$\frac{1}{n}$ = m^{th} term

$\Rightarrow \frac{1}{n}$ = a + (m - 1) d ...(i)

$\frac{1}{m}$= nth term

$\Rightarrow \frac{1}{m}$= a + (n - 1) d ...(ii)

On subtracting equation (ii) from equation (i), we get

$\frac{1}{n} - \frac{1}{m}$ = [a+ (m-1) d] -[a+ (n -1)d]

= a + md - d - a - nd + d

$= (m - n)d$

$\Rightarrow \frac{m-n}{mn} = (m - n)d$

$\Rightarrow d = \frac{1}{mn}$

Putting d = $\frac{1}{mn}$ in equation (i), we get

$\frac{1}{n} = a + \frac{(m-1)}{mn}$

$\Rightarrow \frac{1}{n} = a + \frac{1}{n} - \frac{1}{mn}$

$\Rightarrow a = \frac{1}{mn}$

$\therefore$ $(mn)^{th}$ term = a + (mn - 1) d

= $\frac{1}{mn} + (mn - 1)\frac{1}{mn}$ $\left[\because a = \frac{1}{mn} = d\right]$

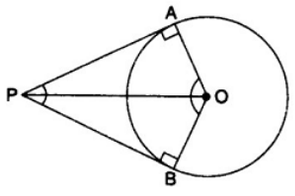
= $\frac{1}{mn} + \frac{mn}{mn} - \frac{1}{mn}$

= 1

29. $\angle OAP = 90^o$ .....(1) [Angle between tangent and radius through the point of contact is 90^o]

$\angle OBP = 90^o$ .....(2) [Angle between tangent and radius through the point of contact is 90^o]

$\therefore$ OAPB is quadrilateral



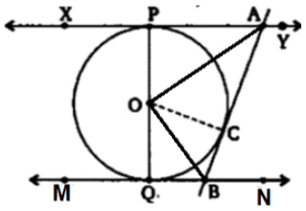
$\therefore \angle APB + \angle AOB + \angle OAP + \angle OBP = 360^o$ [Angle sum property of a quadrilateral]

$\Rightarrow \angle APB + \angle AOB + 90^o + 90^o = 360^o$ [From (1) and (2)]

$\Rightarrow \angle APB + \angle AOB = 180^o$

$\Rightarrow \angle APB$ and $\angle AOB$ are supplementary

OR



To prove $\angle AOB = 90^o$

Proof-

In △AOP and △AOC,

OA = OA [Common]

OP = OC [Both radii]

AP = AC [tangents from external point A]

Therefore by SSS congruence △AOP ≅ △AOC

and by CPCT, $\angle PAO = \angle OAC$

$\Rightarrow \angle PAC = 2\angle OAC$ ...(i)

Similarly, from △OBC and △OBQ, we get;

∠QBC = 2∠OBC ...(ii)

Adding eq. (i) and eq. (ii)

∠PAC + ∠QBC = 180

2(∠OBC + ∠OAC) = 180

(∠OBC + ∠OAC) = 90

Now, in ∠OAB,

Sum of interior angle is 180

So, ∠OBC + ∠OAC + ∠AOB = 180

∴ ∠AOB = 90

hence proved.

30. Given: cosec A + cot A = m

$\Rightarrow$ (cosec A + cot A)^2 = (m)^2 [squaring both sides]

$\Rightarrow$ cosec^{2}A + cot^{2}A + 2 cosec A cot A = m^2 .....(1)

Now, LHS

$= \frac{m^2-1}{m^2+1}$

$= \frac{cosec^2 A+\cot^2 A+2\, cosecA\cot A-1}{cosec^2 A+\cot^2 A+2\, cosec A\cdot\cot A+1}$. [From (1)]

$= \frac{\cot^2 A+\cot^2 A+2\, cosecA\cdot\cot A}{cosec^2 A+\, cosec^2 A+2\, cosecA\cdot\cot A}$ [Since, Cosec^{2}A - Cot^{2}A = 1]

$= \frac{2\cot^2 A+2\, cosecA\cot A}{2\, cosec^2 A+2\, cosecA\cot A}$

$= \frac{2\cot A(\cot A+\, cosecA)}{2\, cosecA(\, cosecA+\cot A)}$

$= \frac{\cot A}{cosecA}$

$= \frac{\frac{\cos A}{\sin A}}{\frac{1}{\sin A}}$

$= \frac{\cos A}{\sin A} \times \frac{\sin A}{1}$

= cos A = RHS

Hence, Proved.

31.

Income	No. of Families	c.f.
$0-1000$	250	250
$1000-2000$	190	$250+190=440$
$2000-3000$	100	$440+100=540$
$3000-4000$	40	$540+40=580$
$4000-5000$	15	$580+15=595$
$5000-6000$	5	$595+5=600$

Here, N = 600

$\Rightarrow$ Median $= \frac{N}{2}$ th term

$= \frac{600}{2} = 300$th term

So, Median class = 1000 - 2000

$l = 1000,\ h = 1000,\ c.f. = 250,\ f = 190$

Median = $l + \left(\frac{\frac{N}{2} - c.f}{f}\right) \times h$

Median = $1000 + \left(\frac{300-250}{190}\right) \times 1000$

= 1000 + $\frac{50}{190} \times 1000$

= 1000 + $\frac{5000}{19}$

$= 1000 + 263.16$

$= 1263.16$

Median = Rs 1263.16

Section D

32. Let the length of piece be x m

Then , rate = $\frac{200}{x}$ per m

Now , new length = (x + 5)m

Since, the cost remains same .

$\therefore$ New rate = $\frac{200}{x+5}$ per m .

Then, $\frac{200}{x+5} = \frac{200}{x} - 2$

$\frac{200}{x+5} = \frac{200-2x}{x}$

$\Rightarrow$ 200x = (x + 5) (200 - 2x)

$\Rightarrow$ 200x = 200x - $2x^2$ + 1000 - 10x

$\Rightarrow$ $2x^2$ + 10x - 1000 = 0

$\Rightarrow$ x^2 + 5x - 500 = 0

$\Rightarrow$ x^2 + 25x – 20x – 500 =0

$\Rightarrow$ x(x + 25) – 20(x +2 5) = 0

$\Rightarrow$ (x – 20) (x + 25) = 0

Therefore, x = 20 or x = -25

But length cannot be negative, therefore x = 20 m

Therefore , length of the piece = 20m

OR

Let the time taken by smaller diameter tap be x hrs.

Time taken by larger diameter tap is (x - 2) hrs.

Therefore $\frac{1}{x-2} + \frac{1}{x} = \frac{8}{15}$

$\Rightarrow$ 15(2x - 2) = 8x(x - 2)

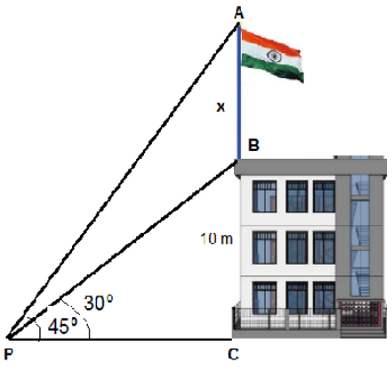
$\Rightarrow$ $8x^2$ - 46x + 30 = 0

$\Rightarrow$ $4x^2$ - 23x + 15 = 0

$\Rightarrow$(4x - 3)(x - 5) = 0

$\Rightarrow$ x = $\frac{3}{4}$, x = 5

33.



As per shown in the figure let BC is the building and AB is flag-staff.

Let AB = x

Then in $\triangle PBC$

$tan\ 30 = \frac{BC}{CP}$

$\frac{1}{\sqrt{3}} = \frac{BC}{CP}$

$CP = BC\sqrt{3}\ = 10\sqrt{3}$ ..... (i)

Now in $\triangle PAC$

$\tan\ 45 = \frac{AC}{CP}$

$1 = \frac{AC}{CP}$

AC = CP

AB + BC = CP

x + 10 = $10\sqrt{3}$

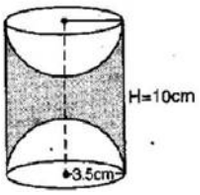
x = $10\sqrt{3}$ - 10 = 10 $\times$1.732 - 10 = 17.32 - 10 = 7.32

So height of flag staff = 7.32 m

and from eqn (1),

CP = $10\sqrt{3}$ = 10 $\times$ 1.732 = 17.32 m

So the distance of point P from the building = 17.32 m

34. 

TSA of the article = $2\pi rh + 2(2\pi r^2)$

= 2π (3.5)(10) + 2[2π $(3.5)^2$]

= $70\pi + 49\pi$

= 119 π

= 119 $\times$ $\frac{22}{7}$

= 374 cm^2

OR

Given,

Radius of cone = Radius of hemisphere = r = 5 cm

Height of cone (h) = 10 cm

No. of cones = 7

Volume of ice cream in one cone = Volume of cone + Volume of hemisphere

= $\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

= $\frac{\pi}{3}r^2(h + 2r)$

= $\frac{22}{7} \times \frac{1}{3} \times 5 \times 5(10 + 2 \times 7)$

= $\frac{22}{7} \times \frac{1}{3} \times 5 \times 5(10 + 10)$

= $\frac{22 \times 25 \times 20}{21}$

= 523.8 cm^3

Volume of ice cream in 7 cones

= 523.8 $\times$ 7 cm^3

= 3666.63 cm^3

= 3.67 litre

35. We make the classes exclusive:

Class Interval	**Frequency f_i**	**Mid value x_i**	**$f_i x_i$**	**Cumulative frequency**
10.5 - 15.5	2	13	26	2
15.5 - 20.5	3	18	54	5
20.5 - 25.5	6	23	138	11
25.5 - 30.5	7	28	196	18

30.5 - 35.5	14	33	462	32
35.5 - 40.5	12	38	456	44
40.5 - 45.5	4	43	172	48
45.5 - 50.5	2	48	96	50
	$\sum f_i = 50$		$\sum f_i x_i = 1600$	

Mean:

Mean = $\frac{\sum f_i x_i}{\sum f_i} = \frac{1600}{50} = 32$

Median:

N= 50 $\Rightarrow \frac{N}{2} = 25$

The cumulative frequency just greater than 25 is 32.

Hence, median class is 30.5 - 35.5

$l = 30.5, h = 5, f = 14, cf = 18$

we know that, Median = $l + \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\}$

$= 30.5 + \left\{ 5 \times \frac{25-18}{14} \right\}$

$= 30.5 + 2.5 = 33$

Section E

36. i. x + y + 2 = 15

x + y = 13 ...(i)

Area of bedroom + Area of kitchen = 95

5 × x + 5 × x + 5 × y = 95

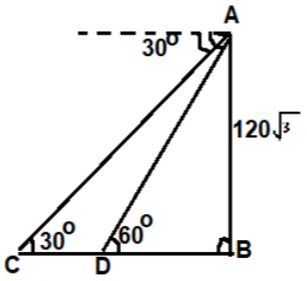
2x + y = 19 ...(ii)

In △ABD

tan 60^o = $\frac{120\sqrt{3}}{BD}$

BD = $\frac{120\sqrt{3}}{\sqrt{3}}$

BD = 120 m



In △ABC

tan 30^o = $\frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{120\sqrt{3}}{BC}$

BC = 360 m

∴ CD = BC - BD

= 360 - 120

= 240 m

ii. Length of outer boundary

= 12 + 15 + 12 + 15

= 54 m

iii.

$$\begin{array}{r} x + y = 13 \\ 2x + y = 19 \\ - \quad - \quad - \\ \hline -x \quad = -6 \end{array}$$

x = 6

Area of bedroom 1 = 5 × x

$= 5 \times 6 = 30\ m^2$

OR

Area of living room = $(5 \times 2) + (9 \times 7)$

$= 10 + 63$

$= 73\ m^2$

37. i. Figures A and C are similar.

ii. Only Figure C is congruent.

iii. All congruent figures are similar but all similar figures are not congruent.

For example, a pair of triangles that are similar by the A.A.A. test of similarity are not congruent pairs of triangles since the exact lengths of the sides are unknown.

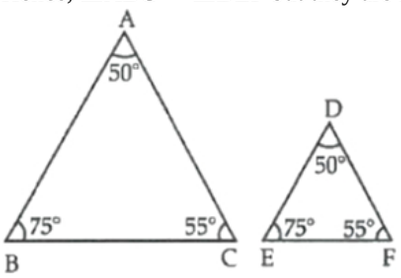
In $\triangle ABC$ and $\triangle DEF$,

$\angle A = \angle D = 50^o$,

$\angle B = \angle E = 75^o$

and $\angle C = \angle F = 55^o$.

Hence, $\triangle ABC \sim \triangle DEF$ but they are not congruent.



OR

The length of corresponding sides must be equal.

38. i. Consider the house is at origin (0, 0), then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, - 8) and (-6, 8).

Since, grocery store is at (2, 3) and food cart is at (6, -8)

$\therefore$ Required distance = $\sqrt{(6-2)^2 + (-8-3)^2}$

$= \sqrt{4^2 + 11^2} = \sqrt{16 + 121} = \sqrt{137}$ cm

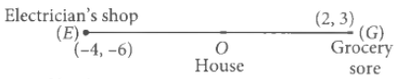
ii. Consider the house is at origin (0, 0), then coordinates of the grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, - 8) and (-6, 8).

Required distance

$= \sqrt{(-6)^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ cm

iii. Consider the house is at origin (0, 0), then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, - 8) and (-6, 8).

Let O divides EG in the ratio k: 1, then



$0 = \frac{2k-4}{k+1}$

$\Rightarrow 2k = 4$

$\Rightarrow k = 2$

Thus, O divides EG in the ratio 2 : 1

Hence, required ratio = OG : OE i.e., 1 : 2.

OR

Consider the house is at origin (0, 0), then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, - 8) and (-6, 8).

Since, (0, 0) is the mid-point of (-6, 8) and (6, -8), therefore both bus stand and food cart are at equal distances from the house.

Hence, required ratio is 1 : 1.